Modal Analysis Based on the Random Decrement Technique

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APPLICATION TO CIVIL ENGINEERING STRUCTURES

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Abstract

This article describes the work carried out within the project: Modal Analysis Based on the Random Decrement Technique - Application to Civil Engineering Structures. The project is part of the research programme Dynamics of Structures sponsored by the Danish Technical Research Council. The planned contents and the requirement for the project prior to its start are described together with the results obtained during the 3 year period of the project. The project was mainly carried out as a Ph.D.-project by the first author from September 1994 to August 1997 in cooperation with associate professor Rune Brincker from the above department. During the project there has been intensive cooperation with professor Sam Ibrahim, Old Dominion University, Virginia, USA. The article is finished with a reference list of the papers and reports prepared during the project. A short description together with the possibility to download papers and reports can be seen at http://www.civil.auc.dk/ĩ6jca

1 Introduction

The project *Modal Analysis Based on the Random Decrement Technique* has mainly been carried out as a Ph.D.-project by the first author. An explanation for the need of investigating the topics of this project can be given by considering the overall topic *Identification of Civil Engineering Structures.* The assignment of identifying civil engineering structures can be solved using several different methods. One of these methods, the ARMAV-model has been investigated as another project within this framework. This method is a very accurate but also a very time consuming method. The Random Decrement (RD) technique is expected to be perhaps the fastest technique of all but less accurate compared to the ARMAV model. So by investigating ARMAV-models and the RD technique both ends of a time/accuracy scale characterizing identification of civil engineering structures have been investigated. Furthermore, the RD technique has been investigated intensively during the late 1980s and early 1990s at the above department mainly by prof. Brincker. The theory of the technique was improved and the practical application of the technique was considered. From the obtained knowledge it was natural to formulate a project concerning the RD technique at Aalborg University in order to take advantage of and finish earlier work.

During this project cooperation with different persons from universities and companies besides prof. Ibrahim have been carried out. The Queensborough bridge, Vancouver, Canada has been identified from ambient data. The data were collected by prof. Ventura and Dr. Felber from

University of Vancouver, Canada. The results from different researchers applying different algorithms to the data were presented and compared at the 14th International Modal Analysis Conference, Dearborn, Michigan, USA, 1996. The author has spent 4 months at the Aristotle University of Thessaloniki, Greece as part of this project. Furthermore, the initial part of a demonstration project concerning the application of vibration based inspection of bridges has been carried out in cooperation with RAMBOLL, Nørresundby, Denmark and the Danish Road Directorate.

2 Plan for the Project

The following topics for the project were described prior to its start.

- Implementation of the RD technique.
- Investigation of the estimation of frequency response functions.
- Investigation of the theoretical background.
- Implementation of modal parameter extraction procedures.

In the next sections it will be pointed out how the different topics and problems have been investigated. It is not the intention to give a detailed mathematical description in this paper. Only the problem, the assumptions, the results and the consequences are described. A detailed report is given in Asmussen [1].

Section 3 contains a short description of the modelling of the vibrations of civil engineering structures and the modelling of the loads of civil engineering structures. Furthermore methods to extract modal parameters from free decays or correlation functions are mentioned. Section 4 gives a brieg description of the RD technique as wll as the progress made during the project. Section 5 describes the Vector Random Decrement technique developed during this project. Section 6 describes a new method to predict the variance of RD functions. Section 7 contains a comparison of the RD and the FFT algorithm for estimation of frequency response functions. The diffent structures and the analysis of the structures using the RD technique are described in section 8 . The implementations performed during this project are described in section 9 and the paper is finished with a conclusion.

3 Civil Engineering Structures

If the aim of a vibration test of a structure is to identify the dynamic characteristics of the structure, the following three processes should all be carried out with high accuracy in order to make the test successful.

- The mathematical modelling of the structural vibrations.
- Measurements of the vibrations.
- Data analysis of the measurements.

These three processes are in practice coherent, so that a separation followed by an individual solution and performance is not possible. This is illustrated by fig. 1, where it is indicated that

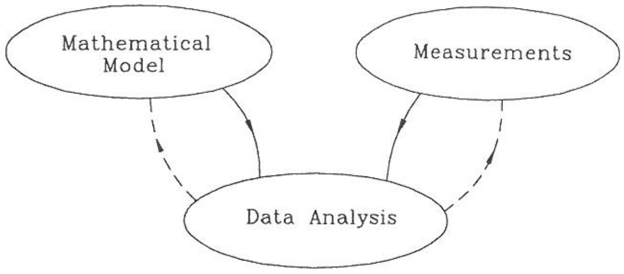

Figure 1: *Illustration of the processes in vibration testing. The data analysis may result in a reformulation of the mathematical model or a change in the experimental setup.*

results of the data analysis could lead to a change of the mathematical model or a change in the experimental setup.

This project deals with the data analysis procedure in vibration testing. This is the process where the measurements of the vibrations of a structure are used to calibrate or identify the parameters of the mathematical model of the structure. The parameters of the mathematical model describe the dynamic characteristics of the structure. Collecting the measurements of the vibrating structure is the fundamental basis for a succesful test. If the measurements are not collected carefully, it is impossible to obtain satisfactorily accurate dynamic characteristics of the structure, regardless of the mathematical modelling and data analysis.

Civil engineering structures are continuous or distributed systems. The mass, damping and stiffness properties are distributed throughout the spatial definition of the structure. In this project the mathematical model used to describe the vibrations of a structure is a discrete model: The linear lumped mass parameter model. The damping forces, which model all energy dissipation from the structure, are assumed to be proportional to the velocity of the lumped masses, and the stiffness forces are assumed to be proportional to the displacements of the lumped masses. The principle is illustrated in fig. 2.

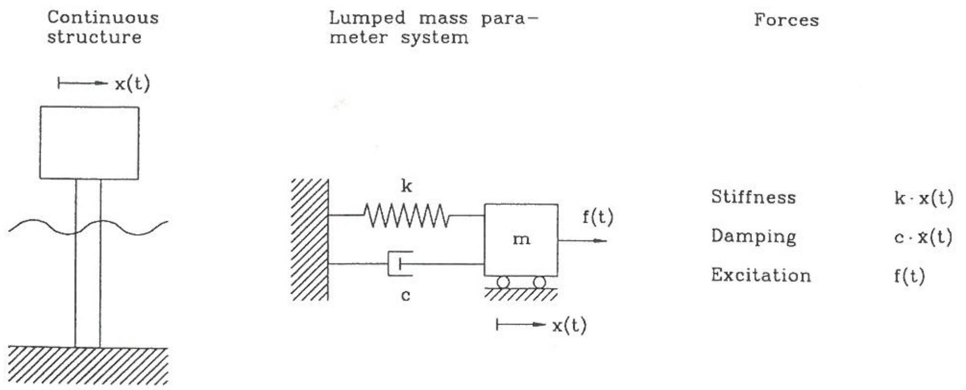

Figure 2: *Example of a continuous structure modelled by a lumped mass parameter system. The displacements at the top of the monopile offshore platform are modelled by a lumped mass parameter system. The loads on the structure, wind and waves, are modelled by $f(t)$, the damping forces are modelled by $c \cdot \dot{x}(t)$ and the stiffness forces are modelled by $k \cdot x(t)$.*

In principle any material point of the continuous structure has a mass, damping and stiffness property attached. So a structure should intuitively have infinitely many of such dynamic degrees of freedom. Since the structure is assumed to be linear, the properties of the dynamic degrees of freedom, which are local properties, can be converted to the dynamic characteristics called modal parameters, which are global parameters of the structure. The modal parameters consist of eigenfrequencies, damping ratios and mode shape vectors. Sometimes the term structural mode or just mode is used. A structural mode is described by an eigenfrequency, damping ratio and a mode shape vector, which belongs together. An eigenfrequency can be interpreted as a frequency where the structure will have a high response level to a harmonic force with that frequency. The eigenfrequencies of a structure are therefore sometimes also denoted resonant frequencies. A damping ratio gives information about how fast the vibrations of the structure at the corresponding eigenfrequency dissipate. The mode shape vectors describe the relative displacements of the material points of the structure at the corresponding eigenfrequency. These properties make the modal parameters global parameters.

Theoretically structures have infinitely many modes and thereby inifinitely many modal parameters. In vibration testing the number of modes, that can be identified, are limited by the force. The forces work in a limited frequency area, so only structural modes with an eigenfrequency within that frequency area are brought into vibration and can therefore be identified. This also means that the lumped mass parameter model of the structure can also be limited. It does not have to contain an infinite number of degrees of freedom. These are the arguments for using the linear lumped mass parameter model with a finite number of masses and thereby finite number of degrees of freedom. This is the mathematical model used to describe the vibrations of the structures considered in the present thesis.

The above assumptions of the mathematical modelling of the structure might seem very restrictive. But it is a very general mathematical model and has been used to model various structures from large civil engineering structures such as high-rise buildings, bridges, offshore platforms to small mechanical systems. It is important that the mathematical model assumes the dynamic characteristics of the structures to be time-invariant during the measurement period. This is also widely assumed. The loads of the structure are assumed to be stationary and Gaussian distributed.

For a general n DOF system the equations of motion become

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) &= \mathbf{f}(t) \\ \mathbf{x}(0) = \mathbf{x}_0, \quad \dot{\mathbf{x}}(0) &= \dot{\mathbf{x}}_0 \end{aligned} \tag{1}$$

This expresses a force equilibrium between external and internal forces. All matrices have the dimension $n \times n$ and the response vectors and force vector have the dimension $n \times 1$, where n is the number of DOFs. $\mathbf{M}$ is a diagonal and positive definite mass matrix. The stiffness matrix, $\mathbf{K}$, is symmetric and positive definite. The dissipation of energy is modelled by the symmetric and positive semi-definite damping matrix $\mathbf{C}$. As indicated in eq. (1) the system is assumed time-invariant, since $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ do not depend on time. The acceleration, velocity and displacement response are denoted $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$, respectively.

The displacements, velocity and acceleration response of the lumped mass parameter system to initial conditions become

$$\mathbf{x}(t) = \mathbf{\Phi} e^{\mathbf{\Lambda} t}\mathbf{q}_0 \tag{2}$$

$$\dot{\mathbf{x}}(t) = \mathbf{\Phi} e^{\mathbf{\Lambda} t}\mathbf{\Lambda}\mathbf{q}_0 \tag{3}$$

$$\ddot{\mathbf{x}}(t) = \mathbf{\Phi} e^{\mathbf{\Lambda} t}\mathbf{\Lambda}^2\mathbf{q}_0 \tag{4}$$

where $\mathbf{\Phi}$ are the mode shapes and $\mathbf{\Lambda}$ contains the eigenvalues from which the eigenfrequencies, f, and damping ratios, ζ, of the system can be extracted, and $\mathbf{q}_0$ contains the initial conditions. As seen, the difference between the free decay displacement response, velocity response and acceleration response is only a complex scaling factor in the form of the eigenvalue matrix, $\mathbf{\Lambda}$. This scaling factor changes the amplitude and the phase of the exponentially damped sinusoidal free decay response. The relations are important, since the modal parameters can be extracted from all free decay responses in eqs. (2) - (4) using algorithms like Ibrahim Time Domain or Polyreference Time Domain, which have been implemented as part of this project.

The load which excites the lumped mass parameter system is assumed to be a stationary zero mean Gaussian distributed vector process. In order to generalize the load process it is assumed that the load process can be described as a white noise vector process passed through a linear shaping filter. This is an extension of the traditional white noise assumption. The idea behind this approach and the theory are presented in Ibrahim et al. [4], where the filter is referred to as a pseudo-force filter. The principle is shown in figure 3.

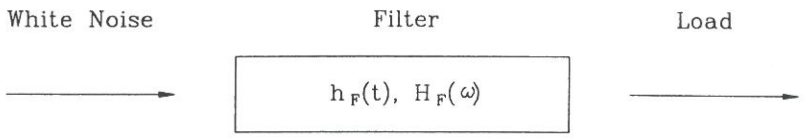

Figure 3: *Outline diagram for modelling of loads using a shaping filter. $h(t)$ and $H(\omega)$ are the impulse response and frequency response matrix, respectively.*

The correlation functions of a stationary stochastic vector process are defined as

$$\mathbf{R}_{\mathbf{XX}}(\tau) = E[\mathbf{X}(t+\tau)\mathbf{X}^T(t)] \tag{5}$$

From this definition and if the above modelling of the structure and the loads is assumed any column in the correlation matrix of $\mathbf{X}$, $\dot{\mathbf{X}}$ or $\ddot{\mathbf{X}}$ can be written as

$$\mathbf{R}^i_{\mathbf{XX}}(\tau) = \tilde{\mathbf{\Phi}} e^{\mathbf{\Lambda}\tau}\tilde{\mathbf{m}}^{-1}\tilde{\mathbf{q}}_i \tag{6}$$

$$R^i_{\dot{\mathbf{X}}\dot{\mathbf{X}}}(\tau) = -\tilde{\mathbf{\Phi}} e^{\mathbf{\Lambda}\tau}\mathbf{\Lambda}^2\mathbf{c}_i \tag{7}$$

$$R^i_{\ddot{\mathbf{X}}\ddot{\mathbf{X}}}(\tau) = \tilde{\mathbf{\Phi}} e^{\mathbf{\Lambda}\tau}\mathbf{\Lambda}^4\mathbf{c}_i \tag{8}$$

where $\mathbf{R}^i_{\mathbf{XX}}$ is an abbreviation of the ith column in the correlation matrix and the scaling constant $\tilde{\mathbf{q}}_i$ is the ith column of the matrix $\tilde{\mathbf{c}}$. Equations (6) - (8) correspond exactly to eqs. (2) - (4), so in the modal parameter extraction procedure there is no reason to distinguish between the correlation functions of the displacements, velocities or the accelerations of the structure or free decays.

4 The Random Decrement Technique

Four major contributions to the theory and application of the RD technique have been prepared

- Generalization of the RD technique by introduction of the general applied triggering condition.
- Introduction of a new concept: Quality assessment.
- Optimal choice of triggering level.
- Comparison of the RD technique with other unbiased methods for estimation of correlation functions.

In the following sections these three topics will be briefly described.

4 .1 The General Applied Triggering condition

The RD technique is a method which transforms the stochastic processes $X(t)$ and $Y(t)$ into RD functions. It is assumed that $X(t)$ and $Y(t)$ are stationary and Gaussian distributed processes with zero mean. The index t is interpreted as time. The RD functions are defined as the mean value of a stochastic process, $Y(t)$ on condition, $T_{X(t)}$

$$D_{YX}(\tau) = E[Y(t+\tau)|T_{X(t)}] \tag{9}$$

An RD function is referred to as e.g. $D_{YX}(\tau)$. The first subscript refers to the process from which the mean value is calculated and the second subscript refers to the process where the condition is fulfilled. The condition $T_{X(t)}$ is denoted triggering condition.

In practical applications of the RD technique only a single realization of the stochastic process is available. Or in other words, usually only a single measurement at each chosen location of a vibrating structure is collected. In order to estimate the conditional mean value correctly from a single observation it is necessary to assume that the stochastic process is not only stationary but also ergodic. In this case the RD functions can be estimated as the empirical conditional mean value from a single realization

$$\hat{D}_{YX}(\tau) = \frac{1}{N} \cdot \sum_{i=1}^{N} y(t_i+\tau)|T_{x(t_i)} \tag{10}$$

where N is the number of points in the process which fulfils the triggering condition and $y(t)$ and $x(t)$ are realizations of $X(t)$ and $Y(t)$. The absolute decisive variable in estimation of RD functions is the number of triggering points, N. N has to be large enough to ensure that eq. (10) has converged sufficiently towards eq. (9).

During this project the triggering condition has been generalized to the applied general triggering condition, $T_{X(t)}^{G_A}$

$$T_{X(t)}^{G_A} = \{a_1 \leq X(t) < a_2, b_1 \leq \dot{X}(t) < b_2\} \tag{11}$$

Using this general triggering condition the RD functions become a weighted sum of the correlation function and the time derivative of the correlation functions

$$D_{YX}(\tau) = \frac{R_{YX}(\tau)}{\sigma_X^2} \cdot \tilde{a} - \frac{R'_{YX}(\tau)}{\sigma_{\dot{X}}^2} \cdot \tilde{b} \tag{12}$$

$$\tilde{a} = \frac{\int_{a_1}^{a_2} x p_X(x) dx}{\int_{a_1}^{a_2} p_X(x) dx} \qquad \tilde{b} = \frac{\int_{b_1}^{b_2} \dot{x} p_{\dot{X}}(\dot{x}) d\dot{x}}{\int_{b_1}^{b_2} p_{\dot{X}}(\dot{x}) d\dot{x}} \tag{13}$$

where the triggering levels $\tilde{a}$ and $\tilde{b}$ are functions of the triggering bounds and the density functions, p. Equation (12) illustrates how versatile the RD technique is. By adjusting the triggering bounds a_1, a_2 and/or b_1, b_2 the contribution of the correlation functions and the time derivative of the correlation functions to the resulting RD functions can be changed. In the limit by choosing one of the triggering level sets, $[a_1\ a_2]$ or $[b_1\ b_2]$, to $[-\infty\ \infty]$ or $[0\ 0]$ the resulting RD functions become proportional to either the correlation functions or their time derivatives.

In application of the RD technique only special formulations of the applied general triggering condition is used. The most common are: Level crossing, $T^L_{X(t)}$, local extremum, $T^E_{X(t)}$, positive point, $T^P_{X(t)}$, zero crossing, $T^Z_{X(t)}$,

$$T^L_{X(t)} = \{X(t) = a\} \tag{14}$$

$$T^E_{X(t)} = \{a_1 \leq X(t) < a_2, \dot{X}(t) = 0\} \tag{15}$$

$$T^P_{X(t)} = \{a_1 \leq X(t) < a_2\} \tag{16}$$

$$T^Z_{X(t)} = \{X(t) = 0, \dot{X}(t) > 0\} \tag{17}$$

The link between the RD functions and the correlation functions can be established directly from the result of eqs. (12) and (13) by inserting the triggering levels. This illustrates the importance of $T^{G_A}_{X(t)}$. The derivation is given in Asmussen [1].

4 .2 Quality Assessment

In application of the RD technique it is an advantage to have some standard methods to assess the quality of the estimated RD functions. The advantage of standard methods is that valuable experience with this technique can be transferred between different data sets obtained from different physical systems. The purpose of quality assessment of RD functions is to answer the following questions.

- Has the empirical mean value converged sufficiently towards the true mean value (or: Is the number of triggering points adequate)?
- How many points in the RD functions are estimated with sufficient accuracy?

Two different types of tests or investigations are suggested as standard methods for quality assessment of RD functions: Shape invariance test and symmetry test.

Testing the shape invariance of RD functions is based on several different estimations of a correlation function using different triggering levels for the same triggering condition

$$R^1_{YX}(\tau) = \frac{D_{YX}(\tau)}{a_1}\sigma^2_X\ , \quad R^2_{YX}(\tau) = \frac{D_{YX}(\tau)}{a_2}\sigma^2_X\ , \quad \ldots \tag{18}$$

where superscripts $1, 2, \ldots$ refer to the different choice of triggering levels. These correlation functions can be compared graphically or the correlation functions between them could be calculated. Any correlation which differs significantly from the others is estimated with erroneous triggering levels.

The second approach suggested for quality assessment of the RD functions is based on the symmetry relations for correlation functions of stationary stochastic processes. This approach generally assumes that all possible RD functions are estimated, corresponding to estimating the full correlation matrix of the measurements at each time step. The approach can still be used if only a part of the correlation matrix is estimated, but this is considered to be a special case. The symmetry relation is

$$R_{YX}(\tau) = R_{XY}(-\tau) \tag{19}$$

From this symmetry relation an average and an error RD function can be calculated by subtraction and addition. From these functions different error functions can be calculated and used as indicator of the optimal choice of triggering level.

Examples and a detailed description are given in Asmussen [1] and Asmussen et al. [15].

4 .3 Optimal Choice of Triggering Level

One of the difficulties in applications of the RD technique is how to choose the triggering levels $[a_1\ a_2]$ for a given triggering condition. From a user point of view it is important to know how to choose a proper triggering level and to know how sensitive the results are to the choice of triggering level. The optimal choice of triggering level is defined as the choice which minimizes the variance of the RD functions normalized to be equal to the correlation functions

$$\min(\mathrm{Var}[\frac{\hat{D}_{XX}(\tau)}{\tilde{a}} \cdot \sigma_X^2]) \rightarrow [a_1\ a_2] \tag{20}$$

For the level triggering condition eq. (20) has been solved. The solution is a triggering level of $a = \sqrt{2}\sigma_X$. The assumptions are that the processes are stationary zero mean Gaussian distributed and that the time segments used in the averaging process are independent. The latter assumption is of course violated in some sense. However the above result is considered a good basis for selecting the triggering level for the level crossing triggering condition.

For the local extremum triggering condition eq. (20) has been investigated for an SDOF system. The optimal triggering levels are shown in fig. 4.

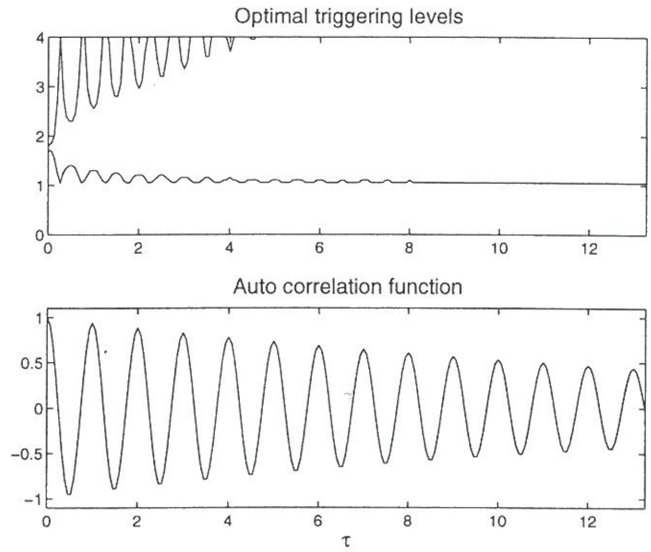

Figure 4: *Theoretically predicted optimal lower and upper triggering level ($\times\sigma_X$) for an SDOF system using local extremum triggering and the corresponding correlation function.*

Figure 4 illustrates a new result. The optimal choice of triggering levels is not always those which maximize the number of triggering points. Theoretically, it is predicted that the triggering levels should be chosen as $[a_1\ a_2] = [\sigma_X\ \infty]$. The above result can only be a guideline to the user, since the result is dependent on the correlation function. Other triggering conditions have been investigated and the general result was that the triggering points between 0 and σ_X should not be used. The work is presented in Asmussen [1] and Asmussen [14].

4 .4 Unbiased Estimation of Correlation Functions

The purpose of this investigation is to compare the RD technique with two other unbiased methods for estimation of correlation functions: The direct approach and an FFT-IFFT approach. The results are presented in Asmussen et al. [6] and Asmussen [1]. The comparison of the different techniques is based on simulations of the displacement response of a 2DOF system loaded by independent white noise at each mass.

The result was that the RD technique is clearly the fastest technique for short correlation functions and as accurate as the FFT-IFFT approach. This indicates that the RD technique can replace the FFT-IFFT technique with advantage.

5 The Vector Random Decrement Technique

A new concept is introduced: Vector triggering Random Decrement (VRD). The argumentation for developing this technique is that experience has discovered some problems with the RD technique applied to a large number of measurements (e.g. >4) collected simultaneously. This is often the situation in ambient testing of bridges.

- If all measurements are used as the reference measurement, which will result in the maximum number of RD functions (full correlation matrix estimated), the estimation time will increase proportionally to the number of measurements compared to the situation where only a single measurement is used as reference measurement. Since the speed is one of the main advantages of this technique a decision of estimating all possible RD setups should be reconsidered. On the other hand, if not all possible RD functions are estimated, valuable information can be lost.

- If only some of the RD setups are estimated, how should the actual reference measurements be chosen. By assuming the signal-to-noise ratio to be highest at the measurements with highest standard deviation, the standard deviation could be used as a criterion. But this is only a guideline, not a full solution.
- Usually the uncertainty of the cross RD functions is higher than the uncertainty of the auto RD functions. So auto RD functions should only be used for modal parameter estimation. But this excludes the possibility to estimate mode shapes.

The above problems are the motivation for developing the VRD technique. The target is somehow to estimate equivalent auto RD functions, which contains phase information and thus the possibility to estimate mode shapes, in contrast to auto RD functions only. The solution is to apply a vector triggering condition instead of a scalar triggering condition. The VRD concept is first presented in Ibrahim et al. [9], [11] and Asmussen et al. [9].

Consider an n-dimensional stochastic vector process, $\mathbf{X}(t)$. The VRD functions are defined equivalently to traditional RD functions

$$\mathbf{D}_{\mathbf{X}}(\tau) = E[\mathbf{X}(t+\tau)|T^v_{\mathbf{X}(t+\mathbf{\Delta t})}] \tag{21}$$

where the vector triggering condition, $T^v_{\mathbf{X}(t+\mathbf{\Delta t})}$, is defined as

$$T^v_{\mathbf{X}(t+\mathbf{\Delta t})} = T^v_{X_1(t+\Delta t_1), X_2(t+\Delta t_2), \ldots, X_m(t+\Delta t_m)} , \quad 2 \leq m \leq n \tag{22}$$

The size of the vector triggering condition has to be in between 2 and n. If the condition is scalar, a traditional RD triggering condition is formulated. A triggering point is detected if several of the elements (m) in the stochastic vector process fulfil individually formulated triggering conditions at any time t plus the individual time shifts Δt_i, $\mathbf{\Delta t} = [\Delta t_1, \Delta t_2, ..., \Delta t_m]$.

The VRD functions are estimated as the empirical mean by assuming the processes to be ergodic

$$\hat{\mathbf{D}}_{\mathbf{X}}(\tau) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{X}(t_i+\tau)|T^v_{\mathbf{X}(t_i+\mathbf{\Delta t})} \tag{23}$$

Estimation of the VRD functions using eq. (23) provides unbiased estimates, corresponding to the RD technique.

Consider an $n \times 1$-dimensional stochastic vector process, $\mathbf{X}(t)$

$$\mathbf{X}(t) = [X_1(t),\ X_2(t),\ \ldots, X_n(t)]^T \tag{24}$$

The vector positive point triggering condition is

$$T^v_{\mathbf{X}(t+\mathbf{\Delta t})} = \{a_1 \leq X_1(t+\Delta t_1) < b_1, \ \ldots \ \{a_m \leq X_m(t+\Delta t_m) < b_m\} \tag{25}$$

The VRD functions of the process using the vector positive point triggering condition become

$$D_{\mathbf{X}}^{v}(\tau) = \mathbf{R}_{X_1}(\tau - \Delta t_1) \cdot \tilde{a}_1 + \ldots \mathbf{R}_{X_m}(\tau - \Delta t_m) \cdot \tilde{a}_m \quad (26)$$

where the triggering level $\tilde{\mathbf{a}}$ is now defined as

$$\tilde{\mathbf{a}} = \frac{\mathbf{R}_{\mathbf{Y}_v \mathbf{Y}_v}^{-1}}{k_1} \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{y}_v \cdot p(\mathbf{y}_v) d\mathbf{y}_v \; k_1 = \int_{\mathbf{a}}^{\mathbf{b}} p_{\mathbf{Y}_v}(\mathbf{y}_v) d\mathbf{y}_v \; \mathbf{Y}_v = [X_1(t) \ldots X_m(t)] \quad (27)$$

and $\mathbf{R}_{X_i}$ is the ith column of the correlation matrix of $\mathbf{X}$. Only the weights, $\tilde{a}_i$, of the correlation functions have changed. In practice only the vector positive point triggering condition is of interest, since this is the only triggering condition, which results in a reasonable number of triggering points.

In conclusion the VRD functions are a sum of a number of correlation functions corresponding to the size of the vector condition. In Ibrahim et al. [8], [10] and Asmussen [9] the technique is validated and the theory is derived in Asmussen [1].

6 Variance of Random Decrement Functions

A new method for estimating the variance of the RD functions is introduced. It will be assumed that the measurements are realizations of stationary zero mean Gaussian distributed processes. A new method should fulfil the following demands.

- The method should be valid for all triggering conditions. This means that the correlation between the time segments should be taken into account and that the method is independent of the chosen triggering condition.
- The method should be accurate and consistent. Consistent means that the accuracy of the method should be independent of the physical system describing the measured responses.
- The method should be fast, otherwise the absolute main advantage of the RD technique is wasted and the method will only have theoretical interest.
- The requirement for the speed of the method is fulfilled if the variance can be predicted by the RD functions only.

In the following a new method is proposed and it is investigated if it fulfils the above demands. Only cross RD functions and the variance of the estimated cross RD functions are considered, in order to simplify the derivations and still preserve generality.

$$\begin{aligned}
\mathrm{Var}[\hat{D}_{YX}(\tau)] = \; & \frac{1}{N^2} \cdot \Bigg(\sum_{i=1}^{N} \mathrm{Cov}[Y(t+\tau)|T_{X(t)}^{G_T}; Y(t+\tau)|T_{X(t)}^{G_T}] \\
& + \sum_{j=1}^{m} N_j \mathrm{Cov}[Y(t+\tau)|T_{X(t)}^{G_T}; Y(t+j\Delta T+\tau)|T_{X(t+j\Delta T)}^{G_T}] \\
& + \sum_{j=1}^{m} N_j \mathrm{Cov}[Y(t+j\Delta T+\tau)|T_{X(t+j\Delta T)}^{G_T}; Y(t+\tau)|T_{X(t)}^{G_T}] \Bigg) \quad (28)
\end{aligned}$$

where $T^{G_T}_{X(t)}$ is of the same form as the theoretical general triggering condition.

$$T^{G_T}_{X(t)} = \{X(t) = a, \dot{X}(t) = b\} \tag{29}$$

The major problem is to calculate the general covariance between $Y(t+\tau)|T^{G_T}_{X(t)}$ and $Y(t+j\Delta T+\tau)|T^{G_T}_{X(t+j\Delta T)}$. Consider the following two Gaussian distributed stochastic vector processes.

$$\mathbf{X}_1 = [Y(t+\tau)\ Y(t+t_1+\tau)]^T \tag{30}$$

$$\mathbf{X}_2 = [X(t)\ X(t+t_1)\ \dot{X}(t)\ \dot{X}(t+t_1)]^T \tag{31}$$

The covariance of $\mathbf{X}_1$ on condition of $\mathbf{X}_2$ is

$$Cov[\mathbf{X}_1|\mathbf{X}_2] = \mathbf{R}_{\mathbf{X}_1\mathbf{X}_1} - \mathbf{R}_{\mathbf{X}_1\mathbf{X}_2}\mathbf{R}^{-1}_{\mathbf{X}_2\mathbf{X}_2}\mathbf{R}^{T}_{\mathbf{X}_1\mathbf{X}_2} \tag{32}$$

Using the definition of the correlation functions in eq. (5) the correlation matrices in (32) can be calculated from eqs. (29) - (30) ($\mathbf{X}$ and $\mathbf{Y}$ are assumed to have zero mean value).

The estimate of the variance of the RD functions involves the following computational steps.

- Sampling the time for each triggering point in estimation of the RD functions.
- Sorting the time differences between the triggering points.
- Numerical (two-time) differentiation of the RD functions (scaled to be the correlation functions).
- Calculating the variance estimate according to eq. (28).

None of these computational steps are extremely time consuming. The sampling of the time points for each triggering point is free, since these time points are identified in the estimation process of the RD functions.

Different simulation studies have been performed and the new approach is superior with respect to accuracy compared to the traditional methods. The method is introduced in Asmussen [1] and Asmussen et al. [2].

7 Estimation of Frequency Response Functions

A new method for estimating FRF has been tested. The method is based on the RD functions of the load to and the response from a linear system. It is assumed that the input is stochastic and stationary. Traditionally the measured response of and load on a linear system have been analysed using the FFT algorithm in order to obtain the FRFs. As described in the introduction to this thesis such an approach will in general always result in biased estimates of the FRM. Using the RD technique as basis a for the estimation of the FRFs bias can under the right circumstances be removed. This is an important feature of the RD technique. This method was first tested in Brincker et al. [8] and Asmussen et al. [3].

By introducing the Fourier transform of an RD function as $Z_{XY}(\omega)$ defined as

$$Z_{XY}(\omega) \doteq \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau} D_{XY}(\tau)d\tau \tag{33}$$

the following frequency domain relation can be proven

$$Z_{YX}(\omega) = H(\omega)Z_{XX}(\omega) \tag{34}$$

and

$$Z_{YY}(\omega) = H(\omega)Z_{XY}(\omega) \tag{35}$$

These two equations constitute a basis for estimating $H(\omega)$. The approach in eq. (34) is denoted H_1^{RD} and the approach in eq. (35) is denoted H_2^{RD}

$$H(\omega) = H_1^{RD} = \frac{Z_{YX}(\omega)}{Z_{XX}(\omega)} \tag{36}$$

$$H(\omega) = H_2^{RD} = \frac{Z_{YY}(\omega)}{Z_{XY}(\omega)} \tag{37}$$

The main advantage of this approach compared to the traditional approach based on pure FFT is that the natural decay of the RD functions makes it possible to avoid bias or leakage.

This is illustrated in fig. 5, where a theoretical FRF of an SDOF system is plotted together with an FFT based estimate and an RD-FFT based estimate. The advantage of unbiased estimates is obvious.

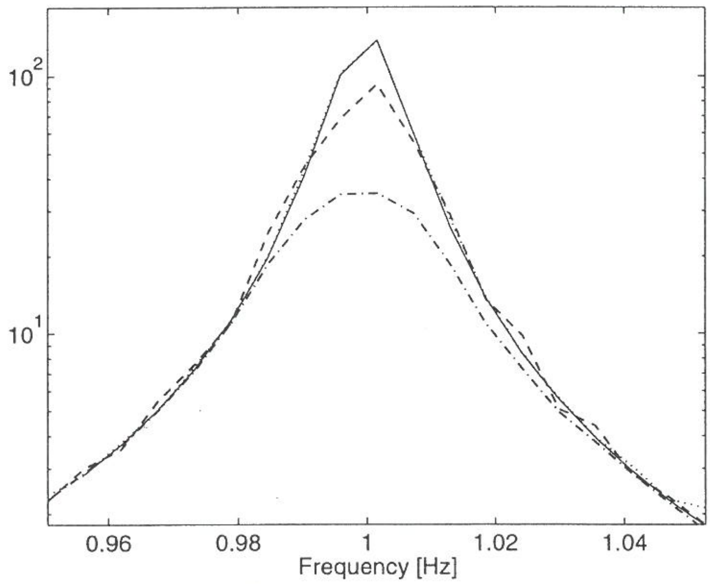

Figure 5: *Zoom of absolute value of the FRFs. [——]: Theoretical. [- - - -]: FFT Hanning window, H_1. [· · · · ·]: RD-FFT, No windowing, H_1^{RD}. [· — · — ·]: RDD-FFT, exponential window, H_1^{RD}.*

8 Application of the Random Decrement Technique to Civil Engineering Structures

The main advantages of the RD technique is the low estimation time and the simple estimation algorithm. The advantage of a low estimation time can be utilized in identification of large structures, where the response of the structure is collected at many locations. In general this is the situation in ambient testing of bridges. Ambient testing of bridges refers to measurements of the vibrations of bridges due to ambient loads such as traffic, wind, waves and micro tremors.

During this project three different bridges have been analysed: The Queensborough bridge, a laboratory bridge model and the Vestvej bridge.

The Queensborough bridge crosses the Fraser river near Vancouver B.C., Canada. The bridge has a length of 200 m and has 3 spans. An outline draft of the bridge is shown in figure 6.

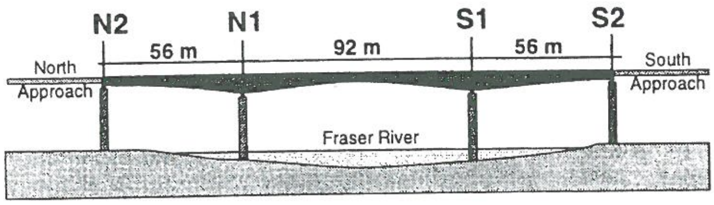

Figure 6: *Outline draft of the Queensborough bridge.*

The analysis of the Queensborough bridge is presented in Asmussen et al. [5].

The purpose of analysing the laboratory bridge model is to make the final development and documentation of the VRD technique. The bridge model is a 3-span simply supported 0.01 m thick steel plate with a total length of 3 m and a width of 0.35 m. Figure 7 illustrates the laboratory bridge model.

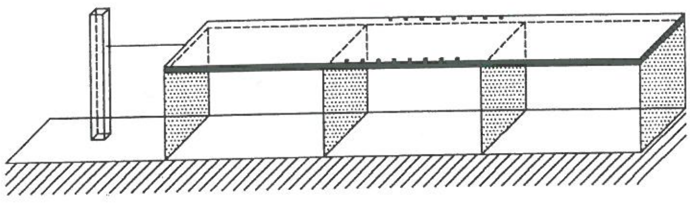

Figure 7: *Laboratory bridge model and sensor locations.*

The analysis of the bridge model is presented in Asmussen et al. [10].

The ambient vibration study of the Vestvej bridge forms the initial part of a demonstration project concerning the application of vibration based inspection to bridges. The Vestvej bridge is shown in fig. 8. The project is carried out with RAMBOLL and the Danish Road Directorate.

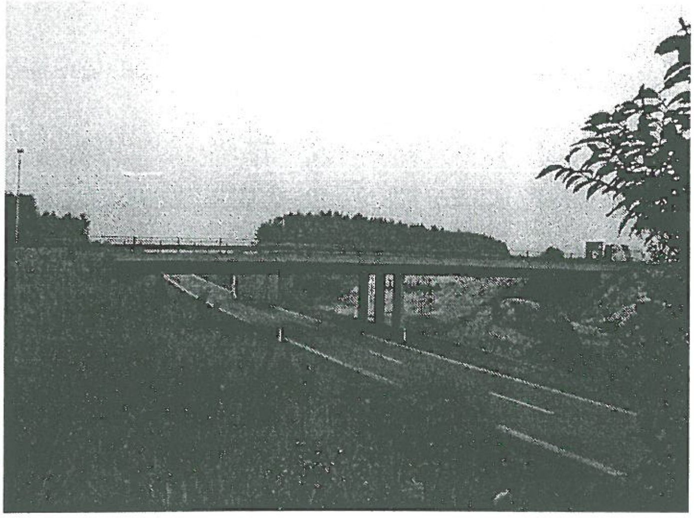

Figure 8: *The Vestvej bridge.*

The analysis of the Vestvej bridge is presented in Asmussen et al. [12], [13], [14] and [15]. The measuremenents of the Vestvej bridge were collected using a bridge measurement system developed as a part of this project.

9 Implementation of the Random Decrement Technique

During the project period different functions have been programmed in MATLAB. These functions are currently being improved and generalized in MATLAB. A Graphical User Interface is under construction.

The programming is expected finished at the end of 1997. A user manual is going to be published with the examples of identification of civil engineering structures performed during this project. The finished program will be presented at the adress http://www.civil.auc.dk/ i6jca.

10 Conclusions

During this project identification of civil engineering structures using the Random Decrement technique has been performed. The theory of the Random Decrement has been extended by the following developments

- Introduction of the general applied triggering condition including the derivation of the relation between the RD functions and correlation functions.
- Introduction of the Vector triggering Random Decrement technique. Derivation of the theoretical background of this approach in terms of correlation functions and validation of the technique by simulation and experimental studies.
- Introduction of the idea, theory and validation of a new approach for predicting the variance of Random Decrement functions.

The practical application of the Random Decrement has also been improved by

- Introduction of a new concept: Quality assessment.
- New guidelines for the choice of triggering levels.
- Comparison with other unbiased estimators of correlation functions.

The technique has been tested on several different structures:

- The Queensborough bridge.
- A laboratory bridge model.
- The Vestvej bridge.

Furthermore, a bridge measurement system has been developed as a part of this project and a MATLAB graphical user interface for identification of ambient excited structures is going to be finished 1/1 1998.

It is concluded that the project more than fulfils the expectations prior to the start of the project.

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